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OPERATING CHARACTERISTIC ANALYSIS FOR
RELIABILITY GROWTH PROGRAMS

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AUGUST 1992

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CONTENTS

	Page
ACKNOWLEDGEMENTS	iii
LIST OF FIGURES	vi
1. INTRODUCTION	1
2. BACKGROUND	1
3. RELIABILITY GROWTH OPERATING CHARACTERISTIC (OC) ANALYSIS..	5
4. APPLICATION	10
5. SUMMARY	14
REFERENCES	15
APPENDIX A - TABLES	17
TABLE FOR 70 PERCENT CONFIDENCE	21
TABLE FOR 80 PERCENT CONFIDENCE	35
TABLE FOR 90 PERCENT CONFIDENCE	49
APPENDIX B - DERIVATIONS	63
DISTRIBUTION LIST	71

LIST OF FIGURES

Figure No.	Title	Page
1	Example OC curve for Reliability Demonstration Test....	4
2	Idealized Reliability Growth Curve	10
3	Program and Alternate Idealized Reliability Growth Curves	11
4	Operating Characteristic (OC) Curve	12

OPERATING CHARACTERISTIC ANALYSIS FOR RELIABILITY GROWTH PROGRAMS

1. INTRODUCTION

The Army Regulation AR-702-3 (Reference 1), entitled Army Materiel Systems Reliability, Availability, Maintainability, mandates that project managers and materiel developers formulate and use reliability growth plans for major and designated non-major developmental systems. These plans are to include reliability growth curves that have been coordinated with the technical independent evaluator.

A well thought out reliability growth plan can serve as a significant management tool in scoping out the required resources to enhance system reliability and demonstrate the system reliability requirement. The principal goal of the growth test is to enhance reliability by the iterative process of surfacing failure modes, analyzing them, implementing corrective actions (fixes), and testing the "improved" configuration to verify fixes and continue the growth process by surfacing remaining failure modes. If the growth test environment during engineering and manufacturing development (EMD) reasonably simulates the mission environment stresses then it may be feasible to use the growth test data to statistically demonstrate the technical, i.e., engineering, requirement (denoted by TR) for system reliability. Such use of the growth test data could eliminate the need to conduct a follow-on reliability demonstration test. The classical demonstration test requires that the system configuration be held constant throughout the test. This type of test is principally conducted to assess and demonstrate the reliability of the configuration under test.

Associated with the demonstration test are statistical consumer and producer risks. In our context, they are frequently termed the Government and contractor risks, respectively. In broad terms, the Government risk is the probability of accepting a system when the true technical reliability is below the TR and the contractor risk is the probability of rejecting a system when the true technical reliability is at least the contractor's target value (set above the TR). An extensive amount of test time may be required for the reliability demonstration test to suitably limit these statistical risks. Moreover, this allotted test time would be principally devoted to demonstrating the system TR associated with the configuration under test instead of to enhancing the system reliability through the reliability growth process of sequential configuration improvement. In today's austere budgetary environment, it is especially important to make maximum use of test resources. With proper planning, a reliability growth program can be an efficient procedure for demonstrating the system reliability requirement while reliability improvements are being achieved via the growth process.

2. BACKGROUND

During a reliability growth test phase, the system configuration is changing due to the activity of surfacing failure modes, analyzing the modes, and implementing fixes to the surfaced modes. It is often reasonable to portray this reliability growth in an idealized manner, i.e., by a smooth rising curve which

captures the overall pattern of growth. The curve relates a measure of system reliability, e.g., mean-time-between-failures (MTBF), to test duration (e.g., hours). The functional form used to express this relationship in MIL-HDBK-189 (Reference 2) is given by

$$M(t) = (M_I / (1 - \alpha)) (t/t_I)^\alpha \quad (1)$$

In this equation, $M(t)$ typically denotes the MTBF achieved after t test hours. The exponent α is termed the growth rate and represents the slope of the assumed linear relationship between $\ln(M(t))$ and $\ln(t)$, where \ln denotes the base e logarithm function. The parameters t_I , M_I may be thought of as defining the initial conditions. In particular, M_I may be interpreted as the MTBF associated with the initial configuration entering the reliability growth test. In this interpretation, t_I would be the planned cumulative test time until one or more fixes are incorporated. An alternate and more general interpretation of M_I and t_I would be to regard M_I as the anticipated average MTBF over an initial test period t_I .

In the above discussion, we have referred to $M(t)$ as the MTBF and have measured test duration by time units, e.g., t hours. We will continue to refer to $M(t)$ and test duration t in this fashion; however, more generally, $M(t)$ may denote mean-miles-to-failure or mean-rounds-to-failure (for a large number of rounds). The corresponding measures of test duration would be test mileage or rounds expended, respectively.

As indicated in Section 1, we shall consider using the data generated during the reliability growth test phase to demonstrate the system reliability technical requirement (TR) at a specified confidence level γ . This paper addresses the case where the data consists of individual failure times $0 < t_1 < t_2 < \dots < t_n \leq T$ for n observed mission reliability failures during test time T , where Equation (1) is assumed to hold for $0 < t \leq T$. Since the MIL-HDBK-189 growth model governed by Equation (1) is being assumed in this paper, we shall also require that the observed number of failures by test duration t , denoted by $N(t)$, be a non-homogeneous Poisson process with intensity function $\rho(t) = \{M(t)\}^{-1}$.

The growth curve planning parameters α, t_I, M_I , and the test time T should be chosen to reasonably limit the consumer (Government) and producer (contractor) statistical risks referred to in Section 1. Prior to presenting the relationship between these risks and the above mentioned parameters, it is instructive to review the determination of these risks for the constant configuration reliability demonstration test.

The parameters defining the reliability demonstration test consist of the test duration T_{dem} , and the allowable number of failures c . Define the random variable F_{obs} to be the number of failures that occur during the test time T_{dem} . Denote the observed value of F_{obs} by f_{obs} . Then the "acceptance" or "passing" criterion is simply $f_{\text{obs}} \leq c$.

Let M denote the MTBF associated with the constant configuration under test. Then F_{obs} has the Poisson probability distribution given by

$$Prob (F_{obs} = i) = e^{-T_{Dem}/M} \frac{(T_{Dem}/M)^i}{i!} \quad (2)$$

Thus the probability of acceptance, denoted by $Prob (A; M, c, T_{Dem})$, as a function of M , c , and T_{Dem} is given by

$$\begin{aligned} Prob(A; M, c, T_{Dem}) &= Prob (F_{obs} \leq c) = \sum_{i=0}^c Prob (F_{obs}=i) \\ &= \sum_{i=0}^c e^{-T_{Dem}/M} \frac{(T_{Dem}/M)^i}{i!} \end{aligned} \quad (3)$$

To ensure "passing the demonstration test" is equivalent to demonstrating the TR at confidence level γ (e.g., $\gamma = 0.80$ or $\gamma = 0.90$), we must choose c such that

$$f_{obs} \leq c \Leftrightarrow TR \leq \ell_\gamma (f_{obs}) \quad (4)$$

where $TR > 0$ and $\ell_\gamma (f_{obs})$ denotes the value of the 100 γ percent lower confidence bound when f_{obs} failures occur in the demonstration test of length T_{Dem} . Note that $\ell_\gamma (f_{obs})$ is a lower confidence bound on the true (but unknown) MTBF of the configuration under test. It is well known (see Proposition 1 in Appendix B) that the following choice of c satisfies (4):

Choose c to be the largest non-negative integer k that satisfies the inequality

$$\sum_{i=0}^k e^{-T_{Dem}/TR} \frac{(T_{Dem}/TR)^i}{i!} \leq 1 - \gamma \quad (5)$$

Note c is well-defined provided

$$\exp (-T_{Dem}/TR) \leq 1 - \gamma \quad (6)$$

Throughout this section we shall assume (6) holds and that c is defined as above.

Recall that the operating characteristic (OC) curve associated with a reliability demonstration test is the graph of the probability of acceptance, i.e., $Prob (A; M, c, T_{Dem})$ given in Equation (3), as a function of the true but unknown constant MTBF M as depicted on Figure 1.

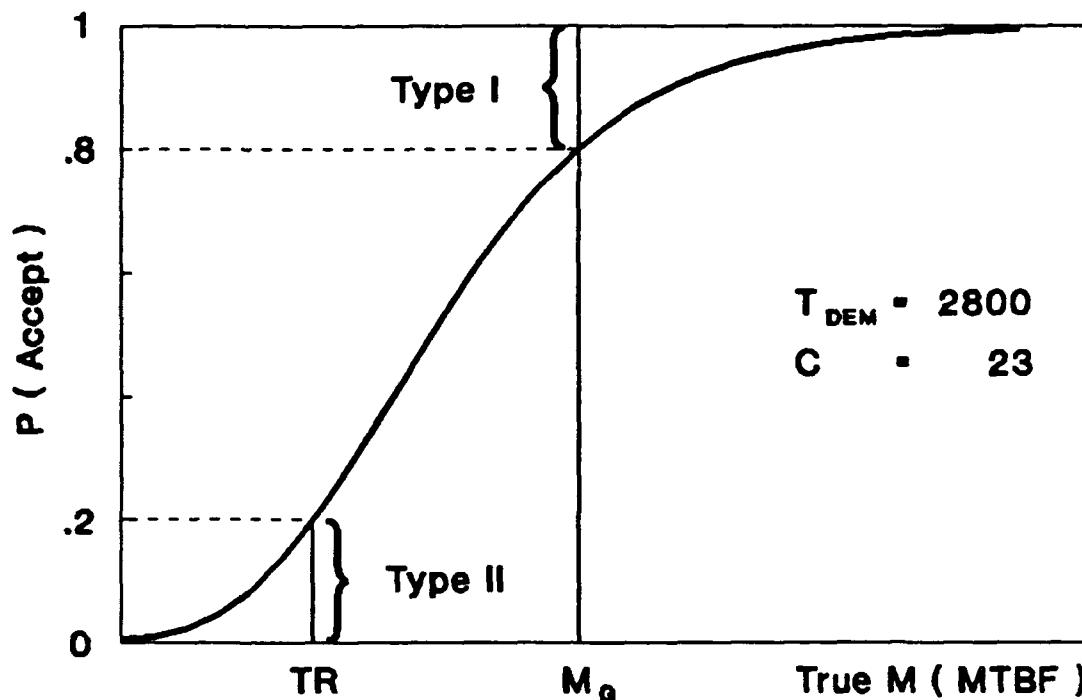


Fig. 1: Example OC Curve for Reliability Demonstration Test

The Government or consumer risk associated with this curve, called the Type II risk, is defined by

$$\text{Type II} \triangleq \text{Prob}(A; TR, C, T_{\text{Dem}}) \quad (7)$$

Thus, by the choice of c ,

$$\text{Type II} \leq 1 - \gamma \quad (8)$$

For the contractor or producer to have a reasonable chance of demonstrating the TR with confidence γ , the system configuration entering the reliability demonstration test must often have a MTBF value, say M_g (the contractor's goal MTBF) that is considerably higher than the TR. The probability that the producer fails the demonstration test given the system under test has a true MTBF value of M_g is termed the producer (contractor) or Type I risk. Thus

$$\text{Type I} = 1 - \text{Prob}(A; M_g, c, T_{\text{Dem}}) \quad (9)$$

If the Type I risk is higher than desired, then either a higher value of M_g should be attained prior to entering the reliability demonstration test or T_{Dem} should be increased. If T_{Dem} is increased then c may have to be readjusted for the new value of T_{Dem} to remain the largest non-negative integer that satisfies inequality (5).

The above numbered equations and inequalities express the relationships between the reliability demonstration test parameters c , T_{Dem} , the requirement parameters TR , γ , and the associated risk parameters (the consumer and producer risks). These relationships are fundamental in conducting tradeoff analyses involving these parameters for planning reliability demonstration tests. In the next section we shall present relationships between the defining parameters for a reliability growth curve (M_i , t_i , α , and T), the requirement parameters (TR and γ), and the associated statistical risk parameters (the consumer and producer risks). Once these relationships are in hand, tradeoffs between these parameters may be utilized to consider demonstrating the TR at confidence level γ by utilizing reliability growth test data.

3. RELIABILITY GROWTH OPERATING CHARACTERISTIC (OC) ANALYSIS

In the previous section, it was noted that for a reliability demonstration test, passing the test could be stated in terms of the allowable number of failures c . It was noted that if c is properly chosen, then passing the test is equivalent to demonstrating the TR at confidence level γ , i.e.,

$$f_{\text{obs}} \leq c \Leftrightarrow TR \leq \ell\gamma (f_{\text{obs}})$$

In the presence of reliability growth, observing c or fewer failures is not equivalent to demonstrating the TR at a given confidence level. The cumulative times to failure as well as the number of failures must be considered when using reliability growth test data to demonstrate the TR at a specified confidence level γ . Thus, the "acceptance" or "passing" criterion must be stated directly in terms of the γ lower confidence bound on $M(T)$ calculated from the reliability growth data. These data will be denoted by (n, s) where n is the number of failures occurring in the growth test of duration T and $s = (t_1, t_2, \dots, t_n)$ is the vector of cumulative failure times. In particular, t_i denotes the cumulative test time to the i^{th} failure and $0 < t_1 < t_2 < \dots < t_n < T$ for $n \geq 1$. We shall also refer to the random vector (N, S) which takes on values (n, s) for $n \geq 1$. Unless otherwise stated, throughout the remainder of this report (N, S) will be conditioned on $N \geq 1$.

Using the lower confidence bound methodology developed for reliability growth data by Crow in Reference 3, we shall define our acceptance criterion by the inequality

$$TR \leq \ell\gamma (n, s) \quad (10)$$

where $\ell\gamma(n, s)$ is the γ statistical lower confidence bound on $M(T)$, calculated as in Reference 3 for $n \geq 1$. Thus, the probability of acceptance is given by

$$\text{Prob} (TR \leq \ell\gamma (N, S)) \quad (11)$$

where the random variable $L_\gamma (N, S)$ takes on the value $\ell\gamma(n, s)$ when (N, S) takes on the value (n, s) .

In accordance with Reference 3, for $n \geq 1$, we define

$$l_{\gamma}(n, s) \triangleq \left(\frac{2n}{z_{\gamma}(n)} \right)^2 \hat{M}_n(T) \quad (12)$$

where $z_{\gamma}(n)$ is the unique positive value of z such that

$$(1/I_1(z)) \sum_{j=1}^n \frac{(z/2)^{2j-1}}{j! (j-1)!} = 1 - \gamma \quad (13)$$

In the above, the function I_1 denotes the modified Bessel function of order one defined as follows:

$$I_1(z) \triangleq \sum_{j=1}^{\infty} \frac{(z/2)^{2j-1}}{j! (j-1)!} \quad (14)$$

In Equation (12), $\hat{M}_n(T)$ denotes the maximum likelihood estimate (MLE) for $M(T)$ given in MIL-HDBK-189 when n failures are observed. As discussed in MIL-HDBK-189,

$$\hat{M}_n(T) = T / (n \hat{\beta}_n) \quad (15)$$

where

$$\hat{\beta}_n = n / \left(\sum_{i=1}^n \ln (T/t_i) \right) \quad (16)$$

The distribution of (N, S) and hence that of $L_{\gamma}(N, S)$ is completely determined by the test duration T together with any set of parameters that define a unique reliability growth curve of the form given by Equation (1) in Section 2. Thus, the value of a probability expression such as given in (11) also depends on T and the assumed underlying growth curve parameters. One such set of parameters, as seen directly from Equation (1), is t_i , M_i , α , together with T . In this growth curve representation, t_i may be arbitrarily chosen subject to $0 < t_i < T$. Alternately, scale parameter $\lambda > 0$ and growth rate α , together with T , can be used to define the growth curve by the equation

$$M(t) = 1 / (\lambda \beta t^{\beta-1}), \quad 0 < t \leq T \quad (17)$$

where $\beta = 1 - \alpha$.

Note by Equation (17),

$$1/\lambda = (M(T)) \beta T^{\beta-1} \quad (18)$$

Thus, the growth curve can also be expressed as

$$M(t) = (M(T)) (t/T)^{\alpha}, \quad 0 < t \leq T \quad (19)$$

By Equation (19) we see that the distribution of (N, S) and hence that of $L_\gamma(N, S)$ is determined by $(\alpha, T, M(T))$.

Unless otherwise stated, throughout the remainder of this report, the distributions for (N, S) and for random variables defined in terms of (N, S) will be with respect to a fixed but unspecified set of values for α , T , $M(T)$ subject only to $\alpha < 1$, $T > 0$, and $M(T) > 0$. The same considerations apply to any associated probability expressions. In particular, the probability of acceptance, i.e., $\text{Prob}(TR \leq L_\gamma(N, S))$, is a function of $(\alpha, T, M(T))$.

To further consider the probability of acceptance, we must first consider several properties of the system of lower confidence bounds generated by $L_\gamma(N, S)$ as specified via Equations (12) through (16). The statistical properties of this system of bounds directly follows from the properties of a set of conditional bounds derived by Crow in Reference 3. These latter bounds are conditioned on a sufficient statistic W which takes on the value

$$w = \sum_{i=1}^n \ln (T/t_i) \quad (20)$$

when (N, S) takes on the value (n, s) .

Let $L_\gamma(N, S; w)$ denote the random variable $L_\gamma(N, S)$ conditioned on $W = w > 0$. In Reference 3 Crow shows that $L_\gamma(N, S; w)$ generates a system of γ lower confidence bounds on $M(T)$, i.e.,

$$\text{Prob}(L_\gamma(N, S; w) \leq M(T)) \geq \gamma \quad (21)$$

for each set of values $(\alpha, T, M(T))$ subject to $\alpha < 1$, $T > 0$, and $M(T) > 0$. Note that the value of w is not known prior to conducting the reliability growth test. Thus, to calculate an OC curve for test planning, i.e., a priori, we wish to base our acceptance criterion on $L_\gamma(N, S)$ as in (11) and not on the conditional random variable $L_\gamma(N, S; w)$. We can utilize Equation (21) to show (see Propositions 2, 3, and 4 in Appendix B) that the Type II or consumer risk for $M(T) = TR$ is at most $1 - \gamma$ (for any $\alpha < 1$ and $T > 0$), analogous to the case in Section 2, i.e.,

$$\text{Type II} = \text{Prob}(TR \leq L_\gamma(N, S)) \leq 1 - \gamma \quad (22)$$

for any $\alpha < 1$ and $T > 0$, provided $M(T) = TR$.

To emphasize the functional dependence of the probability of acceptance on the underlying true growth curve parameters (α , T , $M(T)$), we shall denote this probability by $\text{Prob}(A; \alpha, T, M(T))$. Thus,

$$\text{Prob}(A; \alpha, T, M(T)) \triangleq \text{Prob}(\text{TR} \leq L_\gamma(N, S)) \quad (23)$$

where the distribution of (N, S) and hence that of $L_\gamma(N, S)$ is determined by $(\alpha, T, M(T))$. It can be shown that $\text{Prob}(A; \alpha, T, M(T))$ only depends on the values of $M(T)/\text{TR}$ (or equivalently $M(T)$ for known TR) and $E(N)$. The ratio $M(T)/\text{TR}$ is analogous to the discrimination ratio for a constant configuration reliability demonstration test of the type considered in Section 2. Note $E(N)$ denotes the expected number of failures associated with the growth curve determined by $(\alpha, T, M(T))$. More explicitly, the following equations can be derived (see Propositions 5 and 6 in Appendix B):

$$E(N) = T / \{ (1-\alpha) M(T) \} \quad (24)$$

and

$$\text{Prob}(A; \alpha, T, M(T)) =$$

$$(1 - e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[\text{Prob} \left(\frac{\chi_{2n}^2}{Z_\gamma^2(n)} \geq \frac{1}{2\mu d} \right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right) \quad (25)$$

where $\mu \triangleq E(N)$ and $d \triangleq M(T)/\text{TR}$.

Note (25) shows that the probability of acceptance only depends on μ and d . Thus, we shall subsequently denote the probability of acceptance by $\text{Prob}(A; \mu, d)$.

By (22),

$$\text{Type II} = \text{Prob}(A; \mu, 1) \leq 1 - \gamma \quad (26)$$

Thus, the actual value of the Government or consumer risk solely depends on μ and is at most $1 - \gamma$. To consider the producer or contractor risk, Type I, let α_c denote the contractor's target or goal growth rate. This growth rate should be a value the contractor feels he can achieve for the growth test. Let M_c denote the contractor's MTBF goal. This is the MTBF value the contractor plans to achieve at the conclusion of the growth test of duration T . Thus, if the true growth curve has the parameters α_c and M_c , then the corresponding contractor risk of not demonstrating the TR at confidence level γ (utilizing the generated reliability growth test data) is given by

$$\text{Type I} = 1 - \text{Prob}(A; \mu_c, d_c) \quad (27)$$

where

$$d_G = M_G/TR \text{ and } \mu_G = T/((1-\alpha_G)M_G) \quad (28)$$

If the Type I risk is higher than desired, there are several ways to consider reducing this risk while maintaining the Type II risk at or below $1-\gamma$. Since $\text{Prob}(A; \mu_G, d_G)$ is an increasing function of μ_G and d_G , the Type I risk can be reduced by increasing one or both of these quantities, e.g., by increasing T .

To further consider how the Type I statistical risk can be influenced, we shall express d_G and μ_G in terms of TR , T , α_G , and the initial conditions (M_I, t_I) .

Using Equations (1) and (19) with $\alpha = \alpha_G$ and $M(T) = M_G$, by (28) we can show

$$M_G/TR = d_G = \left\{ \frac{M_I}{(1-\alpha_G) t_I^{\alpha_G} TR} \right\} T^{\alpha_G} \quad (29)$$

and

$$E(N) = \mu_G = (t_I^{\alpha_G}/M_I) T^{1-\alpha_G} \quad (30)$$

Note for a given requirement TR , initial conditions (M_I, t_I) , and an assumed positive growth rate α_G , the contractor risk is a decreasing function of T via Equations (27), (29), and (30). These equations can be used to solve for a test time T such that the contractor risk is a specified value. The corresponding Government risk will be at most $1-\gamma$ and is given by Equation (26).

Section 4 contains two examples of an OC analysis for planning a reliability growth program. The first example illustrates the construction of an OC curve for given initial conditions (M_I, t_I) and requirement TR . The second example illustrates the iterative solution for the amount of test time T necessary to achieve a specified contractor (producer) risk, given initial conditions (M_I, t_I) , and requirement TR . These examples use Equations (29) and (30) rewritten as in Equations (1) and (24), respectively, i.e.,

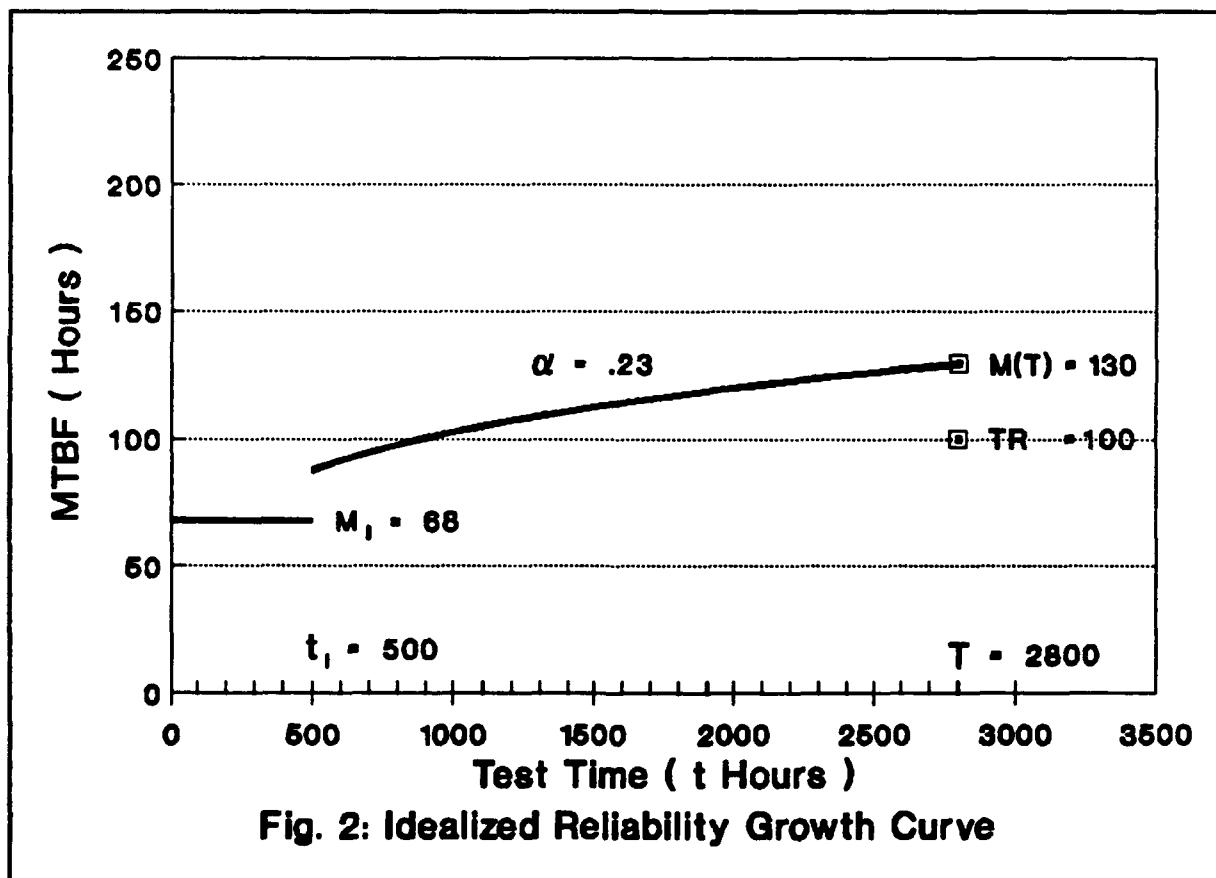
$$M(T) = \left(\frac{M_I}{1-\alpha} \right) \left(\frac{T}{t_I} \right)^{\alpha} \text{ and } E(N) = \frac{T}{(1-\alpha) M(T)}$$

The quantities $d=M(T)/TR$ and $\mu=E(N)$ are then used to obtain an approximation to $\text{Prob}(A; \mu, d)$. Approximate values are provided in Appendix A for a range of values for μ and d . The nature of this approximation is also discussed in Appendix A.

4. APPLICATION

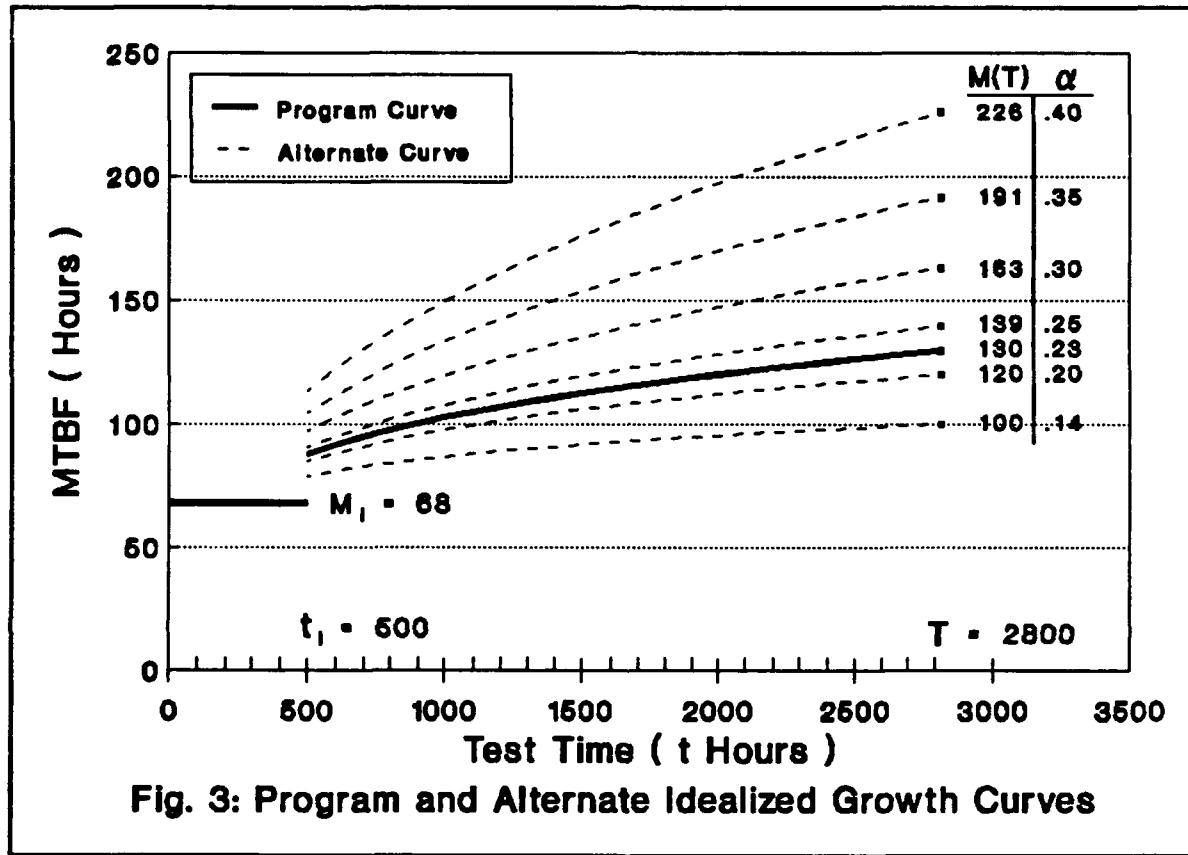
4.1 Example 1.

Suppose we have a system under development which has a technical requirement (TR) MTBF of 100 hours to be demonstrated with 80 percent confidence. For the developmental program, a total of 2800 hours test time (T) at the system level has been predetermined for reliability growth purposes. Based on historical data for similar type systems and on lower level testing for the system under development, the initial MTBF (M_i) averaged over the first 500 hours (t_1) of system-level testing was expected to be 68 hours. Using these data, an idealized reliability growth curve was constructed such that if the tracking curve followed along the idealized growth curve, the TR MTBF of 100 hours would be demonstrated with 80 percent confidence. The growth rate (α) and the final MTBF ($M(T)$) for the idealized growth curve were 0.23 and 130 hours, respectively. The idealized growth curve for this program is depicted on Figure 2.



For this example, suppose we want to determine the operating characteristic (OC) curve for the program. For this, we need to consider alternate idealized growth curves where the $M(T)$ vary but the M_i and t_1 remain the same values as those for the program idealized growth curve; i.e., $M_i = 68$ hours and $t_1 = 500$ hours. In varying the $M(T)$, this is analogous to considering alternate values

of the true MTBF for a reliability demonstration test of a fixed configuration system. For this program, one alternate idealized growth curve was determined where $M(T)$ equals the TR whereas the remaining alternate idealized growth curves were determined for different values of the growth rate α . These alternate idealized growth curves along with the program idealized growth curve are depicted on Figure 3.

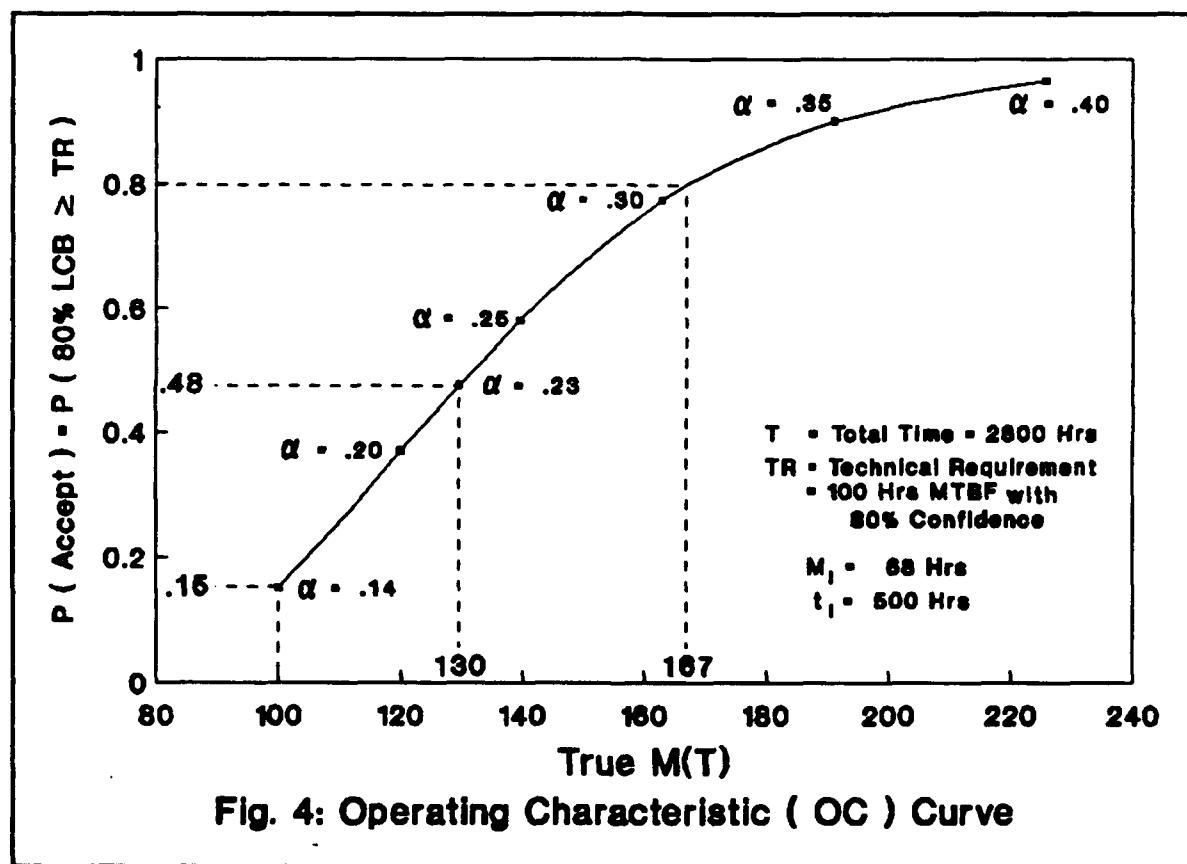


Now, for each idealized growth curve we find $M(T)$ and the expected number of failures $E(N)$ from the equations

$$M(T) = \frac{M_I}{(1-\alpha)} \left(\frac{T}{t_I} \right)^\alpha \text{ and } E(N) = \frac{T}{M(T)(1-\alpha)} .$$

Using the ratio $M(T)/TR$ and $E(N)$ as entries in the tables contained in Appendix A, we determine, by double linear interpolation, the probability of demonstrating the TR with 80 percent confidence. This probability is actually the probability that the 80 percent lower confidence bound (80 percent LCB) for $M(T)$ will be greater than or equal to the TR. These probabilities represent the probability of acceptance ($P(A)$) points on the OC curve for this program which is depicted on Figure 4. The $M(T)$, α , $E(N)$, and $P(A)$ for these idealized growth curves are summarized in the following table:

M(T)	α	E(N)	P(A)
100	0.14	32.6	0.15
120	0.20	29.2	0.37
130	0.23	28.0	0.48
139	0.25	26.9	0.58
163	0.30	24.5	0.77
191	0.35	22.6	0.90
226	0.40	20.6	0.96



From the OC curve, the Type I or producer risk is 0.52 (1-0.48) which is based on the program idealized growth curve where $M(T) = 130$. Note that if the true growth curve were the program idealized growth curve, there is still a 0.52 probability of not demonstrating the TR with 80 percent confidence. This occurs even though the true reliability would grow to $M(T) = 130$ which is considerably higher than the TR value of 100. The Type II or consumer risk, which is based on the alternate idealized growth curve where $M(T) = TR = 100$, is 0.15. As indicated on the OC curve, it should be noted that for this developmental program to have a producer risk of 0.20, the contractor would have to plan on an idealized growth curve with $M(T) = 167$.

4.2 Example 2.

Consider a system under development which has a technical requirement (TR) MTBF of 100 hours to be demonstrated with 80 percent confidence, as in Example 1. The initial MTBF (M_I) over the first 500 hours (t_I) of system level testing for this system was estimated to be 48 hours which, again as in Example 1, was based on historical data for similar type systems and on lower level testing for the system under development. For this developmental program, it was assumed that a growth rate (α) of 0.30 would be appropriate for reliability growth purposes. Now, for this example, suppose we want to determine the total amount of system level test time (T) such that the Type I or producer risk for the program idealized reliability growth curve is 0.20; i.e., the probability of not demonstrating the TR of 100 hours with 80 percent confidence is 0.20 for the final MTBF value ($M(T)$) obtained from the program idealized growth curve. This probability corresponds to the probability of acceptance ($P(A)$) point of 0.80 (1-0.20) on the operating characteristic (OC) curve for this program.

Now, to determine the test time T which will satisfy the Type I or producer risk of 0.20, we first select an initial value of T and, as in Example 1, find $M(T)$ and the expected number of failures ($E(N)$) from the equations

$$M(T) = \frac{M_I}{(1-\alpha)} \left(\frac{T}{t_I} \right)^\alpha \quad \text{and} \quad E(N) = \frac{T}{M(T)(1-\alpha)}$$

Then, again, using the ratio $M(T)/TR$ and $E(N)$ as entries in the tables contained in Appendix A, we determine, by double linear interpolation, the probability of demonstrating the TR with 80 percent confidence. An iterative procedure is then applied until the $P(A)$ obtained from the table equals the desired 0.80 within some reasonable accuracy. For this example, suppose we selected 3000 hours as our initial estimate of T and obtained the following iterative results:

T	M(T)	E(N)	P(A)
3000	117.4	36.5	<0.412
4000	128.0	44.6	<0.610
5000	136.8	52.2	<0.793
5500	140.8	55.8	0.815
5400	140.0	55.1	0.804
5300	139.2	54.4	0.790
5350	139.6	54.7	0.796
5375	139.8	54.9	0.800

Based on these results, we determine $T = 5375$ hours to be the required amount of system level test time such that the Type I or producer risk for the program idealized growth curve is 0.20.

5. SUMMARY

The concepts of an operating characteristic (OC) analysis have been extended to the reliability growth setting. Government (consumer) and contractor (producer) statistical risks have been expressed in terms of the underlying growth curve parameters, test duration, and reliability requirement. In particular, for a given confidence level, these risks have been shown to depend solely on the expected number of failures during the growth test and the ratio of the MTBF to be achieved at the end of the growth program to the MTBF technical requirement to be demonstrated with confidence. Formulas have been developed for computing these risks as a function of the test duration and growth curve planning parameters.

The methodology developed and illustrated in this report should be of interest to RAM analysts responsible for structuring realistic reliability growth programs to achieve and demonstrate program objectives with reasonable statistical risks. In particular, this methodology allows the RAM analysts to construct a reliability growth curve which considers both the Government and contractor risks prior to agreeing to a reliability growth program.

REFERENCES

1. AR 702-3, "Army Materiel Systems Reliability, Availability, Maintainability," April 1989.
2. MIL-HDBK-189, "Reliability Growth Management," 13 February 1981.
3. Crow, Larry H., AMSAA TR-197, "Confidence Interval Procedures for Reliability Growth Analysis," June 1977.
4. Ziad, Tariq and Ellner, Paul, AMSAA TR-453, "Statistical Precision and Robustness of the AMSAA Continuous Reliability Growth Estimators," April 1988.

APPENDIX A
TABLES

18

APPENDIX A TABLES

The following tables provide approximations to the probability of acceptance, Prob (A; μ , d), where μ denotes the expected number of failures and $d = M(T)/TR$. The tabular entries were calculated using a modification to Equation (25). This modification entails (1) approximating $z_\gamma^2(n)$ by $4n\chi_{n+2,\gamma}^2$ and (2) conditioning on $N \geq 2$ instead of $N > 1$. Thus, in Equation (25) the expression $1 - e^{-x}$ is replaced by $1 - P(N \leq 1) = 1 - e^{-\mu} - \mu e^{-\mu}$ and the summation is over $N \geq 2$.

The approximation used for $z_\gamma^2(n)$ follows from the lower confidence bound approximation given by

$$\ell_\gamma(n, s) \approx (n/\chi_{n+2,\gamma}^2) \hat{M}_n \quad (31)$$

where \hat{M}_n is the MLE of $M(T)$ calculated from the observed data $s = (t_1, t_2, \dots, t_n)$. Here t_i denotes the cumulative operating time to the i^{th} failure. This approximation was suggested by Dr. Larry Crow for conveniently approximating $\ell_\gamma(n, s)$. It has been our experience that the approximation in (31) results in slightly more conservative lower bounds on $M(T)$ than $\ell_\gamma(n, s)$. This implies that use of the corresponding approximation to $z_\gamma^2(n)$ would yield slightly smaller values of Prob (A; μ , d) than one would obtain by utilizing $z_\gamma^2(n)$. Based on our experience with Prob (A; μ , d) estimated by simulation, the approximating values appear to be within 0.01 of values obtained through simulation. We also observed that the approximation improves as n increases. The comparison between the lower confidence bound approximation given by (31) and the lower confidence bound using $z_\gamma^2(n)$ was based on Table C-1 contained in Reference 2. Since the entries in this table were for $n \geq 2$, the probability of acceptance, Prob (A; μ , d), was conditioned on $N \geq 2$. In most cases of interest for the model discussed in this report, Prob ($N \geq 2$) will be close to one. In this situation, conditioning on $N \geq 2$ yields values of Prob (A; μ , d) that are, for practical purposes, essentially the same as those obtained by conditioning on $N \geq 1$.

The entries in these tables were calculated using the well-known relationship between the complement of a Chi-square distribution function and the cumulative Poisson sum. This relationship was applied to calculate

$$\text{Prob} \left(\chi_{2n}^2 \geq \frac{z_\gamma^2(n)}{2\mu d} \right)$$

in the expression for $\text{Prob}(A; \mu, d)$ in Section 3 with $z_\gamma^2(n)$ replaced by its approximation, i.e., $4n\chi_{n+2,\gamma}^2$. In terms of the cumulative Poisson sum, this yields

$$\text{Prob} \left(\chi_{2n}^2 \geq \frac{2(n\chi_{n+2,\gamma}^2)}{\mu d} \right) = \sum_{x=0}^{n-1} e^{-w} \frac{w^x}{x!} \quad (32)$$

where

$$w = (n\chi_{n+2,\gamma}^2) / (\mu d).$$

With additional computational effort, one can more precisely calculate $\text{Prob}(A; \mu, d)$ by iteratively solving for $z_\gamma^2(n)$ as the z-solution to Equation (13) of Section 3 over an appropriate range of n . Then Equation (32) can be utilized with $2n\chi_{n+2,\gamma}^2$ replaced by $z_\gamma^2(n)/2$.

The tables contained in this appendix are approximation values of $\text{Prob}(A; \mu, d)$ for three confidence levels; namely, for $\gamma = 0.70$, $\gamma = 0.80$, and $\gamma = 0.90$.

TABLE FOR
70 PERCENT CONFIDENCE

**TABLE FOR
80 PERCENT CONFIDENCE**

**TABLE FOR
90 PERCENT CONFIDENCE**

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	5	6	7	8	9	10	11	12
1.00	0.036	0.044	0.049	0.053	0.055	0.057	0.059	0.060
1.05	0.043	0.052	0.059	0.064	0.067	0.071	0.073	0.076
1.10	0.050	0.062	0.070	0.076	0.081	0.085	0.089	0.093
1.15	0.059	0.072	0.082	0.089	0.096	0.102	0.107	0.112
1.20	0.068	0.083	0.095	0.104	0.113	0.120	0.127	0.134
1.25	0.078	0.095	0.109	0.120	0.130	0.140	0.149	0.157
1.30	0.088	0.108	0.124	0.137	0.150	0.161	0.172	0.182
1.35	0.099	0.121	0.140	0.156	0.170	0.184	0.197	0.209
1.40	0.111	0.136	0.156	0.175	0.192	0.208	0.223	0.238
1.45	0.124	0.151	0.174	0.195	0.214	0.233	0.250	0.267
1.50	0.136	0.166	0.192	0.216	0.238	0.258	0.278	0.298
1.55	0.150	0.183	0.211	0.237	0.262	0.285	0.307	0.329
1.60	0.164	0.199	0.231	0.260	0.287	0.312	0.337	0.361
1.65	0.178	0.217	0.251	0.282	0.312	0.340	0.367	0.393
1.70	0.193	0.234	0.271	0.305	0.337	0.368	0.397	0.425
1.75	0.208	0.252	0.292	0.329	0.363	0.396	0.427	0.456
1.80	0.223	0.270	0.313	0.352	0.389	0.424	0.457	0.488
1.85	0.238	0.289	0.334	0.376	0.415	0.451	0.486	0.518
1.90	0.254	0.307	0.355	0.399	0.440	0.479	0.515	0.548
1.95	0.270	0.326	0.376	0.423	0.465	0.505	0.543	0.578
2.00	0.286	0.345	0.398	0.446	0.490	0.532	0.570	0.606
2.05	0.302	0.364	0.419	0.469	0.515	0.557	0.596	0.633
2.10	0.318	0.382	0.439	0.491	0.539	0.582	0.622	0.658
2.15	0.334	0.401	0.460	0.513	0.562	0.606	0.646	0.683
2.20	0.351	0.419	0.480	0.535	0.585	0.629	0.670	0.706
2.25	0.367	0.437	0.500	0.556	0.606	0.652	0.692	0.729
2.30	0.383	0.456	0.520	0.577	0.628	0.673	0.714	0.749
2.35	0.399	0.473	0.539	0.597	0.648	0.694	0.734	0.769
2.40	0.415	0.491	0.557	0.616	0.668	0.713	0.753	0.787
2.45	0.430	0.508	0.576	0.635	0.686	0.732	0.771	0.805
2.50	0.446	0.525	0.593	0.653	0.705	0.749	0.788	0.821
2.55	0.461	0.541	0.611	0.670	0.722	0.766	0.803	0.835
2.60	0.476	0.558	0.627	0.687	0.738	0.782	0.818	0.849
2.65	0.491	0.573	0.644	0.703	0.754	0.796	0.832	0.862
2.70	0.505	0.589	0.659	0.719	0.769	0.810	0.845	0.874
2.75	0.520	0.604	0.674	0.733	0.783	0.824	0.857	0.885
2.80	0.534	0.618	0.689	0.748	0.796	0.836	0.868	0.895
2.85	0.547	0.633	0.703	0.761	0.809	0.847	0.879	0.904
2.90	0.561	0.646	0.717	0.774	0.821	0.858	0.889	0.913
2.95	0.574	0.660	0.730	0.786	0.832	0.868	0.897	0.920
3.00	0.587	0.673	0.742	0.798	0.842	0.878	0.906	0.927

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	13	14	15	16	17	18	19	20
1.00	0.062	0.063	0.064	0.065	0.066	0.066	0.067	0.068
1.05	0.078	0.080	0.082	0.083	0.085	0.087	0.088	0.090
1.10	0.096	0.099	0.102	0.105	0.108	0.111	0.113	0.116
1.15	0.117	0.121	0.126	0.130	0.134	0.138	0.142	0.146
1.20	0.140	0.146	0.152	0.158	0.164	0.169	0.174	0.180
1.25	0.165	0.173	0.181	0.188	0.196	0.203	0.210	0.217
1.30	0.192	0.202	0.212	0.221	0.230	0.239	0.248	0.257
1.35	0.221	0.233	0.245	0.256	0.267	0.278	0.289	0.300
1.40	0.252	0.266	0.280	0.293	0.306	0.319	0.331	0.344
1.45	0.284	0.300	0.316	0.331	0.346	0.361	0.375	0.389
1.50	0.317	0.335	0.353	0.370	0.387	0.403	0.419	0.435
1.55	0.350	0.370	0.390	0.409	0.428	0.446	0.463	0.480
1.60	0.384	0.406	0.427	0.448	0.468	0.488	0.507	0.525
1.65	0.418	0.442	0.465	0.487	0.508	0.529	0.549	0.568
1.70	0.451	0.477	0.502	0.525	0.548	0.569	0.590	0.610
1.75	0.485	0.512	0.537	0.562	0.585	0.608	0.629	0.649
1.80	0.517	0.546	0.572	0.598	0.622	0.644	0.666	0.686
1.85	0.549	0.578	0.606	0.632	0.656	0.679	0.701	0.721
1.90	0.580	0.610	0.638	0.664	0.689	0.711	0.733	0.753
1.95	0.610	0.640	0.669	0.695	0.719	0.742	0.763	0.782
2.00	0.639	0.669	0.697	0.723	0.747	0.770	0.790	0.809
2.05	0.666	0.696	0.725	0.750	0.774	0.795	0.815	0.833
2.10	0.692	0.722	0.750	0.775	0.798	0.819	0.837	0.854
2.15	0.716	0.746	0.773	0.798	0.820	0.840	0.858	0.873
2.20	0.739	0.769	0.795	0.819	0.840	0.859	0.876	0.891
2.25	0.761	0.790	0.816	0.838	0.858	0.876	0.892	0.906
2.30	0.781	0.809	0.834	0.856	0.875	0.892	0.906	0.919
2.35	0.800	0.827	0.851	0.872	0.890	0.905	0.919	0.930
2.40	0.818	0.844	0.866	0.886	0.903	0.917	0.930	0.940
2.45	0.834	0.859	0.881	0.899	0.915	0.928	0.940	0.949
2.50	0.849	0.873	0.893	0.911	0.925	0.938	0.948	0.957
2.55	0.863	0.886	0.905	0.921	0.935	0.946	0.955	0.963
2.60	0.875	0.897	0.915	0.930	0.943	0.953	0.962	0.969
2.65	0.887	0.908	0.925	0.939	0.950	0.960	0.967	0.974
2.70	0.898	0.917	0.933	0.946	0.957	0.965	0.972	0.978
2.75	0.907	0.926	0.941	0.953	0.962	0.970	0.976	0.981
2.80	0.916	0.934	0.947	0.958	0.967	0.974	0.980	0.984
2.85	0.924	0.941	0.953	0.964	0.972	0.978	0.983	0.987
2.90	0.932	0.947	0.959	0.968	0.975	0.981	0.985	0.989
2.95	0.938	0.952	0.963	0.972	0.979	0.984	0.988	0.991
3.00	0.944	0.958	0.968	0.975	0.981	0.986	0.989	0.992

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T) / TR	21	22	23	24	25	26	27	28
1.00	0.068	0.069	0.070	0.070	0.071	0.071	0.072	0.072
1.05	0.091	0.093	0.094	0.095	0.096	0.098	0.099	0.100
1.10	0.118	0.121	0.123	0.125	0.127	0.130	0.132	0.134
1.15	0.150	0.153	0.157	0.160	0.164	0.167	0.170	0.174
1.20	0.185	0.190	0.195	0.200	0.205	0.209	0.214	0.219
1.25	0.224	0.230	0.237	0.243	0.250	0.256	0.262	0.269
1.30	0.266	0.274	0.282	0.290	0.298	0.306	0.314	0.322
1.35	0.310	0.320	0.330	0.340	0.349	0.359	0.368	0.378
1.40	0.356	0.368	0.379	0.391	0.402	0.413	0.424	0.434
1.45	0.403	0.416	0.429	0.442	0.455	0.467	0.479	0.491
1.50	0.450	0.465	0.479	0.494	0.507	0.521	0.534	0.547
1.55	0.497	0.513	0.528	0.544	0.558	0.573	0.587	0.600
1.60	0.542	0.560	0.576	0.592	0.607	0.622	0.637	0.650
1.65	0.587	0.604	0.621	0.638	0.654	0.669	0.683	0.697
1.70	0.629	0.647	0.664	0.681	0.697	0.712	0.726	0.740
1.75	0.668	0.687	0.704	0.721	0.736	0.751	0.765	0.779
1.80	0.706	0.724	0.741	0.757	0.772	0.787	0.800	0.813
1.85	0.740	0.758	0.774	0.790	0.805	0.818	0.831	0.843
1.90	0.771	0.789	0.805	0.820	0.833	0.846	0.858	0.869
1.95	0.800	0.816	0.832	0.846	0.859	0.871	0.882	0.892
2.00	0.826	0.841	0.856	0.869	0.881	0.892	0.902	0.911
2.05	0.849	0.864	0.877	0.889	0.900	0.910	0.919	0.927
2.10	0.869	0.883	0.896	0.907	0.917	0.926	0.934	0.941
2.15	0.888	0.900	0.912	0.922	0.931	0.939	0.946	0.952
2.20	0.904	0.915	0.926	0.935	0.943	0.950	0.956	0.962
2.25	0.918	0.928	0.938	0.946	0.953	0.959	0.965	0.969
2.30	0.930	0.940	0.948	0.955	0.961	0.967	0.972	0.976
2.35	0.940	0.949	0.957	0.963	0.968	0.973	0.977	0.981
2.40	0.950	0.957	0.964	0.970	0.974	0.978	0.982	0.985
2.45	0.957	0.964	0.970	0.975	0.979	0.983	0.985	0.988
2.50	0.964	0.970	0.975	0.980	0.983	0.986	0.988	0.990
2.55	0.970	0.975	0.980	0.983	0.986	0.989	0.991	0.993
2.60	0.975	0.979	0.983	0.986	0.989	0.991	0.993	0.994
2.65	0.979	0.983	0.986	0.989	0.991	0.993	0.994	0.995
2.70	0.982	0.986	0.989	0.991	0.993	0.994	0.996	0.996
2.75	0.985	0.988	0.991	0.993	0.994	0.996	0.996	0.997
2.80	0.988	0.990	0.992	0.994	0.995	0.996	0.997	0.998
2.85	0.990	0.992	0.994	0.995	0.996	0.997	0.998	0.998
2.90	0.991	0.993	0.995	0.996	0.997	0.998	0.998	0.999
2.95	0.993	0.995	0.996	0.997	0.998	0.998	0.999	0.999
3.00	0.994	0.996	0.997	0.998	0.998	0.999	0.999	0.999

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T) / TR	29	30	31	32	33	34	35	36
1.00	0.072	0.073	0.073	0.074	0.074	0.074	0.075	0.075
1.05	0.101	0.102	0.103	0.104	0.105	0.106	0.107	0.108
1.10	0.136	0.138	0.140	0.142	0.144	0.146	0.147	0.149
1.15	0.177	0.180	0.183	0.186	0.189	0.192	0.195	0.198
1.20	0.223	0.228	0.232	0.237	0.241	0.246	0.250	0.254
1.25	0.275	0.281	0.287	0.292	0.298	0.304	0.310	0.315
1.30	0.329	0.337	0.344	0.352	0.359	0.366	0.373	0.380
1.35	0.387	0.396	0.404	0.413	0.422	0.430	0.439	0.447
1.40	0.445	0.455	0.465	0.475	0.485	0.495	0.504	0.513
1.45	0.503	0.514	0.525	0.536	0.547	0.557	0.568	0.578
1.50	0.559	0.571	0.583	0.595	0.606	0.617	0.628	0.638
1.55	0.613	0.626	0.638	0.650	0.662	0.673	0.684	0.695
1.60	0.664	0.677	0.689	0.701	0.713	0.724	0.735	0.745
1.65	0.711	0.723	0.736	0.748	0.759	0.770	0.780	0.790
1.70	0.753	0.766	0.778	0.789	0.800	0.810	0.820	0.829
1.75	0.791	0.803	0.815	0.825	0.835	0.845	0.854	0.863
1.80	0.825	0.836	0.847	0.857	0.866	0.875	0.883	0.891
1.85	0.854	0.865	0.874	0.883	0.892	0.900	0.907	0.914
1.90	0.880	0.889	0.898	0.906	0.913	0.920	0.927	0.933
1.95	0.901	0.910	0.918	0.925	0.931	0.937	0.943	0.948
2.00	0.919	0.927	0.934	0.940	0.946	0.951	0.956	0.960
2.05	0.935	0.941	0.947	0.953	0.958	0.962	0.966	0.970
2.10	0.947	0.953	0.958	0.963	0.967	0.971	0.974	0.977
2.15	0.958	0.963	0.967	0.971	0.975	0.978	0.980	0.983
2.20	0.967	0.971	0.974	0.978	0.981	0.983	0.985	0.987
2.25	0.973	0.977	0.980	0.983	0.985	0.987	0.989	0.991
2.30	0.979	0.982	0.985	0.987	0.989	0.990	0.992	0.993
2.35	0.984	0.986	0.988	0.990	0.992	0.993	0.994	0.995
2.40	0.987	0.989	0.991	0.992	0.994	0.995	0.996	0.996
2.45	0.990	0.992	0.993	0.994	0.995	0.996	0.997	0.997
2.50	0.992	0.994	0.995	0.996	0.996	0.997	0.998	0.998
2.55	0.994	0.995	0.996	0.997	0.997	0.998	0.998	0.999
2.60	0.995	0.996	0.997	0.998	0.998	0.998	0.999	0.999
2.65	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999
2.70	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.75	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000
2.80	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.85	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.90	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.95	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
3.00	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	37	38	39	40	41	42	43	44
1.00	0.075	0.075	0.076	0.076	0.076	0.077	0.077	0.077
1.05	0.109	0.110	0.111	0.112	0.113	0.113	0.114	0.115
1.10	0.151	0.153	0.155	0.156	0.158	0.160	0.162	0.163
1.15	0.201	0.204	0.207	0.210	0.213	0.215	0.218	0.221
1.20	0.258	0.262	0.267	0.271	0.275	0.279	0.283	0.287
1.25	0.321	0.326	0.332	0.337	0.343	0.348	0.353	0.358
1.30	0.387	0.394	0.401	0.407	0.414	0.421	0.427	0.433
1.35	0.455	0.463	0.471	0.479	0.486	0.494	0.501	0.509
1.40	0.522	0.531	0.540	0.549	0.557	0.566	0.574	0.582
1.45	0.587	0.597	0.606	0.616	0.625	0.633	0.642	0.651
1.50	0.649	0.659	0.668	0.678	0.687	0.696	0.705	0.713
1.55	0.705	0.715	0.724	0.734	0.743	0.752	0.760	0.768
1.60	0.755	0.765	0.774	0.783	0.792	0.800	0.808	0.816
1.65	0.800	0.809	0.818	0.826	0.834	0.842	0.849	0.856
1.70	0.838	0.847	0.855	0.862	0.870	0.877	0.883	0.889
1.75	0.871	0.878	0.886	0.892	0.899	0.905	0.911	0.916
1.80	0.898	0.905	0.911	0.917	0.923	0.928	0.933	0.937
1.85	0.920	0.926	0.932	0.937	0.941	0.946	0.950	0.954
1.90	0.938	0.943	0.948	0.952	0.956	0.960	0.963	0.966
1.95	0.953	0.957	0.961	0.964	0.967	0.970	0.973	0.976
2.00	0.964	0.967	0.971	0.974	0.976	0.978	0.981	0.983
2.05	0.973	0.976	0.978	0.981	0.983	0.984	0.986	0.988
2.10	0.980	0.982	0.984	0.986	0.987	0.989	0.990	0.991
2.15	0.985	0.987	0.988	0.990	0.991	0.992	0.993	0.994
2.20	0.989	0.990	0.992	0.993	0.994	0.994	0.995	0.996
2.25	0.992	0.993	0.994	0.995	0.996	0.996	0.997	0.997
2.30	0.994	0.995	0.996	0.996	0.997	0.997	0.998	0.998
2.35	0.996	0.996	0.997	0.997	0.998	0.998	0.998	0.999
2.40	0.997	0.997	0.998	0.998	0.998	0.999	0.999	0.999
2.45	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.50	0.998	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.55	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.60	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.65	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	45	46	47	48	49	50	51	52
1.00	0.077	0.077	0.078	0.078	0.078	0.078	0.078	0.079
1.05	0.116	0.117	0.117	0.118	0.119	0.120	0.121	0.121
1.10	0.165	0.167	0.168	0.170	0.171	0.173	0.175	0.176
1.15	0.224	0.226	0.229	0.232	0.234	0.237	0.240	0.242
1.20	0.291	0.295	0.298	0.302	0.306	0.310	0.314	0.317
1.25	0.364	0.369	0.374	0.379	0.384	0.389	0.394	0.399
1.30	0.440	0.446	0.452	0.458	0.464	0.470	0.476	0.482
1.35	0.516	0.523	0.530	0.537	0.544	0.551	0.558	0.564
1.40	0.590	0.598	0.605	0.613	0.620	0.627	0.635	0.642
1.45	0.659	0.667	0.675	0.683	0.690	0.698	0.705	0.712
1.50	0.721	0.729	0.737	0.745	0.752	0.759	0.767	0.773
1.55	0.776	0.784	0.792	0.799	0.806	0.813	0.819	0.825
1.60	0.824	0.831	0.838	0.844	0.851	0.857	0.863	0.868
1.65	0.863	0.870	0.876	0.882	0.887	0.892	0.898	0.902
1.70	0.895	0.901	0.906	0.911	0.916	0.921	0.925	0.929
1.75	0.921	0.926	0.931	0.935	0.939	0.943	0.946	0.949
1.80	0.941	0.945	0.949	0.953	0.956	0.959	0.962	0.964
1.85	0.957	0.960	0.963	0.966	0.969	0.971	0.973	0.975
1.90	0.969	0.972	0.974	0.976	0.978	0.980	0.982	0.983
1.95	0.978	0.980	0.982	0.983	0.985	0.986	0.988	0.989
2.00	0.984	0.986	0.987	0.989	0.990	0.991	0.992	0.993
2.05	0.989	0.990	0.991	0.992	0.993	0.994	0.994	0.995
2.10	0.992	0.993	0.994	0.995	0.995	0.996	0.996	0.997
2.15	0.995	0.995	0.996	0.996	0.997	0.997	0.998	0.998
2.20	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999
2.25	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
2.30	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.35	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.40	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

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PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	53	54	55	56	57	58	59	60
1.00	0.079	0.079	0.079	0.079	0.079	0.080	0.080	0.080
1.05	0.122	0.123	0.123	0.124	0.125	0.126	0.126	0.127
1.10	0.178	0.179	0.181	0.182	0.184	0.185	0.187	0.188
1.15	0.245	0.247	0.250	0.252	0.255	0.257	0.260	0.262
1.20	0.321	0.325	0.328	0.332	0.336	0.339	0.343	0.346
1.25	0.404	0.408	0.413	0.418	0.422	0.427	0.432	0.436
1.30	0.488	0.494	0.499	0.505	0.511	0.516	0.522	0.527
1.35	0.571	0.577	0.584	0.590	0.596	0.602	0.608	0.614
1.40	0.649	0.655	0.662	0.669	0.675	0.681	0.687	0.694
1.45	0.719	0.726	0.732	0.739	0.745	0.751	0.757	0.763
1.50	0.780	0.786	0.793	0.799	0.805	0.811	0.816	0.822
1.55	0.832	0.838	0.843	0.849	0.854	0.859	0.864	0.869
1.60	0.874	0.879	0.884	0.889	0.893	0.898	0.902	0.906
1.65	0.907	0.912	0.916	0.920	0.924	0.927	0.931	0.934
1.70	0.933	0.937	0.940	0.943	0.947	0.950	0.952	0.955
1.75	0.953	0.955	0.958	0.961	0.963	0.966	0.968	0.970
1.80	0.967	0.969	0.971	0.973	0.975	0.977	0.979	0.980
1.85	0.977	0.979	0.981	0.982	0.984	0.985	0.986	0.987
1.90	0.985	0.986	0.987	0.988	0.989	0.990	0.991	0.992
1.95	0.990	0.991	0.992	0.992	0.993	0.994	0.994	0.995
2.00	0.993	0.994	0.995	0.995	0.996	0.996	0.996	0.997
2.05	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.998
2.10	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999
2.15	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
2.20	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.25	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

\$ PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	61	62	63	64	65	66	67	68
1.00	0.080	0.080	0.080	0.081	0.081	0.081	0.081	0.081
1.05	0.128	0.128	0.129	0.130	0.130	0.131	0.131	0.132
1.10	0.190	0.191	0.192	0.194	0.195	0.197	0.198	0.200
1.15	0.265	0.267	0.270	0.272	0.274	0.277	0.279	0.281
1.20	0.350	0.353	0.357	0.360	0.364	0.367	0.371	0.374
1.25	0.441	0.445	0.450	0.454	0.459	0.463	0.467	0.472
1.30	0.532	0.538	0.543	0.548	0.553	0.558	0.563	0.568
1.35	0.620	0.626	0.631	0.637	0.642	0.648	0.653	0.659
1.40	0.700	0.705	0.711	0.717	0.722	0.728	0.733	0.739
1.45	0.769	0.775	0.780	0.786	0.791	0.796	0.801	0.806
1.50	0.827	0.832	0.837	0.842	0.847	0.852	0.856	0.860
1.55	0.874	0.878	0.883	0.887	0.891	0.895	0.899	0.902
1.60	0.910	0.914	0.916	0.921	0.924	0.928	0.931	0.934
1.65	0.938	0.941	0.944	0.946	0.949	0.951	0.954	0.956
1.70	0.958	0.960	0.962	0.964	0.966	0.968	0.970	0.972
1.75	0.972	0.974	0.975	0.977	0.978	0.980	0.981	0.982
1.80	0.982	0.983	0.984	0.985	0.986	0.987	0.988	0.989
1.85	0.988	0.989	0.990	0.991	0.991	0.992	0.993	0.993
1.90	0.993	0.993	0.994	0.994	0.995	0.995	0.996	0.996
1.95	0.995	0.996	0.996	0.997	0.997	0.997	0.997	0.998
2.00	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
2.05	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
2.10	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.15	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T) / TR	69	70	71	72	73	74	75	76
1.00	0.081	0.081	0.081	0.082	0.082	0.082	0.082	0.082
1.05	0.133	0.133	0.134	0.135	0.135	0.136	0.136	0.137
1.10	0.201	0.202	0.204	0.205	0.206	0.208	0.209	0.210
1.15	0.284	0.286	0.289	0.291	0.293	0.295	0.298	0.300
1.20	0.377	0.381	0.384	0.387	0.391	0.394	0.397	0.400
1.25	0.476	0.480	0.484	0.488	0.493	0.497	0.501	0.505
1.30	0.573	0.578	0.583	0.588	0.592	0.597	0.602	0.606
1.35	0.664	0.669	0.674	0.679	0.684	0.689	0.694	0.698
1.40	0.744	0.749	0.754	0.759	0.763	0.768	0.773	0.777
1.45	0.811	0.816	0.820	0.825	0.829	0.833	0.837	0.842
1.50	0.865	0.869	0.873	0.877	0.880	0.884	0.888	0.891
1.55	0.906	0.909	0.913	0.916	0.919	0.922	0.925	0.928
1.60	0.937	0.939	0.942	0.944	0.947	0.949	0.951	0.953
1.65	0.958	0.960	0.962	0.964	0.966	0.968	0.969	0.971
1.70	0.973	0.975	0.976	0.978	0.979	0.980	0.981	0.982
1.75	0.983	0.984	0.985	0.986	0.987	0.988	0.989	0.990
1.80	0.990	0.991	0.991	0.992	0.992	0.993	0.993	0.994
1.85	0.994	0.994	0.995	0.995	0.996	0.996	0.996	0.997
1.90	0.996	0.997	0.997	0.997	0.998	0.998	0.998	0.998
1.95	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
2.00	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.05	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T) / TR	77	78	79	80	81	82	83	84
1.00	0.082	0.082	0.082	0.082	0.082	0.083	0.083	0.083
1.05	0.138	0.138	0.139	0.139	0.140	0.141	0.141	0.142
1.10	0.212	0.213	0.214	0.216	0.217	0.218	0.220	0.221
1.15	0.302	0.304	0.307	0.309	0.311	0.313	0.316	0.318
1.20	0.404	0.407	0.410	0.413	0.416	0.419	0.422	0.426
1.25	0.509	0.513	0.517	0.521	0.525	0.528	0.532	0.536
1.30	0.611	0.615	0.620	0.624	0.628	0.633	0.637	0.641
1.35	0.703	0.708	0.712	0.717	0.721	0.725	0.730	0.734
1.40	0.782	0.786	0.790	0.795	0.799	0.803	0.807	0.811
1.45	0.845	0.849	0.853	0.857	0.860	0.864	0.867	0.871
1.50	0.894	0.898	0.901	0.904	0.907	0.910	0.913	0.915
1.55	0.930	0.933	0.935	0.938	0.940	0.942	0.944	0.946
1.60	0.955	0.957	0.959	0.961	0.963	0.964	0.966	0.967
1.65	0.972	0.974	0.975	0.976	0.977	0.979	0.980	0.981
1.70	0.983	0.984	0.985	0.986	0.987	0.988	0.988	0.989
1.75	0.990	0.991	0.991	0.992	0.993	0.993	0.993	0.994
1.80	0.994	0.995	0.995	0.996	0.996	0.996	0.996	0.997
1.85	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998
1.90	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
1.95	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000
2.00	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	85	86	87	88	89	90	91	92
1.00	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
1.05	0.142	0.143	0.143	0.144	0.144	0.145	0.146	0.146
1.10	0.222	0.224	0.225	0.226	0.227	0.229	0.230	0.231
1.15	0.320	0.322	0.324	0.326	0.329	0.331	0.333	0.335
1.20	0.429	0.432	0.435	0.438	0.441	0.444	0.447	0.450
1.25	0.540	0.544	0.547	0.551	0.555	0.558	0.562	0.565
1.30	0.645	0.649	0.653	0.657	0.661	0.665	0.669	0.673
1.35	0.738	0.742	0.746	0.750	0.754	0.758	0.762	0.765
1.40	0.815	0.818	0.822	0.826	0.829	0.833	0.836	0.840
1.45	0.874	0.877	0.880	0.884	0.887	0.889	0.892	0.895
1.50	0.918	0.920	0.923	0.925	0.928	0.930	0.932	0.934
1.55	0.948	0.950	0.952	0.954	0.956	0.957	0.959	0.961
1.60	0.969	0.970	0.971	0.973	0.974	0.975	0.976	0.977
1.65	0.982	0.983	0.984	0.984	0.985	0.986	0.987	0.987
1.70	0.990	0.990	0.991	0.991	0.992	0.992	0.993	0.993
1.75	0.994	0.995	0.995	0.995	0.996	0.996	0.996	0.997
1.80	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998
1.85	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.90	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

M(T)/TR	EXPECTED NUMBER OF FAILURES							
	93	94	95	96	97	98	99	100
1.00	0.084	0.084	0.084	0.084	0.084	0.084	0.084	0.084
1.05	0.147	0.147	0.148	0.148	0.149	0.149	0.150	0.150
1.10	0.232	0.234	0.235	0.236	0.237	0.239	0.240	0.241
1.15	0.337	0.339	0.341	0.343	0.345	0.348	0.350	0.352
1.20	0.453	0.456	0.458	0.461	0.464	0.467	0.470	0.473
1.25	0.569	0.573	0.576	0.580	0.583	0.586	0.590	0.593
1.30	0.677	0.681	0.684	0.688	0.692	0.695	0.699	0.702
1.35	0.769	0.773	0.776	0.780	0.783	0.787	0.790	0.794
1.40	0.843	0.846	0.849	0.852	0.855	0.858	0.861	0.864
1.45	0.898	0.900	0.903	0.905	0.908	0.910	0.913	0.915
1.50	0.936	0.938	0.940	0.942	0.944	0.946	0.948	0.949
1.55	0.962	0.964	0.965	0.966	0.968	0.969	0.970	0.971
1.60	0.978	0.979	0.980	0.981	0.982	0.983	0.983	0.984
1.65	0.988	0.989	0.989	0.990	0.990	0.991	0.991	0.992
1.70	0.994	0.994	0.994	0.995	0.995	0.995	0.996	0.996
1.75	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998
1.80	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
1.85	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

**APPENDIX B
DERIVATIONS**

APPENDIX B
DERIVATIONS

Proposition 1.

$$f_{\text{obs}} \leq c \Leftrightarrow TR \leq \ell_Y(f_{\text{obs}})$$

Proof.

To prove this relation, we use the equation below which follows directly from the definition of a 100 γ percent lower confidence bound when f_{obs} failures occur in a demonstration test of length T_{Dem} :

$$\sum_{i=0}^{f_{\text{obs}}} e^{-T_{\text{Dem}}/t} \frac{(T_{\text{Dem}}/t)^i}{i!} = 1 - \gamma$$

where

$$\ell \triangleq \ell_Y(f_{\text{obs}}).$$

Let g be the function of $x > 0$ defined by the left hand side of the equation above with ℓ replaced by x . Note g is a strictly increasing function of $x > 0$ since $g(x)$ is the probability of obtaining f_{obs} or fewer failures when the constant configuration under test has MTBF x .

I. First we shall show $f_{\text{obs}} \leq c \Rightarrow TR \leq \ell$.

Thus, let $f_{\text{obs}} \leq c$. Suppose $\ell < TR$. Then

$$\begin{aligned} g(\ell) &< g(TR) = \sum_{i=0}^{f_{\text{obs}}} e^{-T_{\text{Dem}}/TR} \frac{(T_{\text{Dem}}/TR)^i}{i!} \\ &\leq \sum_{i=0}^c e^{-T_{\text{Dem}}/TR} \frac{(T_{\text{Dem}}/TR)^i}{i!} \\ &\leq 1 - \gamma, \text{ which is a contradiction since } g(\ell) = 1 - \gamma. \text{ Thus, } TR \leq \ell. \end{aligned}$$

II. Next we shall show $TR \leq \ell \Rightarrow f_{\text{obs}} \leq c$. Thus, let $TR \leq \ell$. Suppose $f_{\text{obs}} > c$. Then

$$\sum_{i=0}^{f_{\text{obs}}} e^{-T_{\text{Dem}}/TR} \frac{(T_{\text{Dem}}/TR)^i}{i} = g(TR) \leq g(t) = 1-\gamma.$$

Since $f_{\text{obs}} > c$, this contradicts the definition of c (see Equation (5) in Section 2). Thus $f_{\text{obs}} \leq c$.

Proposition 2.

For each $\alpha < 1$, $T > 0$, and $M(T) > 0$, the corresponding distribution function of $L_\gamma(N, S)$ satisfies the inequality

$$\text{Prob}(L_\gamma(N, S) \leq M(T)) \geq \gamma$$

Proof

Let f_w denote the density function of W (defined by Equation (20) in Section 3) corresponding to $\alpha < 1$, $T > 0$, $M(T) > 0$. By inequality (21) in Section 3,

$$\begin{aligned} & \text{Prob}(L_\gamma(N, S) \leq M(T)) \\ &= \int_0^\infty \{\text{Prob}(L_\gamma(N, S; w) \leq M(T))\} f_N(w) dw \\ &\geq \gamma \int_0^\infty f_N(w) dw = \gamma \end{aligned}$$

Proposition 3.

For each $\alpha < 1$, $T > 0$, and $M(T) > 0$,

$$\text{Prob}(L_\gamma(N, S) = x) = 0$$

for all real x .

Proof

Let $\alpha < 0$, $T > 0$, and $M(T) > 0$. Clearly $L_\gamma(N, S) \geq 0$. Thus, we need to consider $x \geq 0$.

Let $L_\gamma(n, S)$ denote $L_\gamma(N, S)$ conditioned on $N=n$. As shown in Appendix A of Reference 4,

$$\frac{\hat{M}_n(T)}{M(T)} \sim \left(\frac{\lambda T^\beta}{2n^2} \right) \chi_{2n}^2$$

where χ_v^2 is the chi-square random variable with v degrees of freedom.

Thus,

$$\begin{aligned}\hat{M}_n(T) &\sim \left(\frac{1}{\lambda \beta T^{\beta-1}} \right) \left(\frac{\lambda T^\beta}{2n^2} \right) \chi_{2n}^2 \\ &= \left(\frac{T}{2\beta n^2} \right) \chi_{2n}^2\end{aligned}$$

Then, by (12) in Section 3,

$$L_\gamma(n, S) \sim \left(\frac{2n}{z_\gamma(n)} \right)^2 \left(\frac{T}{2\beta n^2} \right) \chi_{2n}^2,$$

i.e.,

$$L_\gamma(n, S) \sim \left(\frac{2T}{\beta} \right) \left(\frac{\chi_{2n}^2}{z_\gamma^2(n)} \right) \quad (33)$$

Thus,

$$\text{Prob}(L_\gamma(n, S) = x) = \text{Prob}\left(\chi_{2n}^2 = \frac{\beta z_\gamma^2(n)x}{2T}\right) = 0$$

It then follows that,

$$\begin{aligned}\text{Prob}(L_\gamma(N, S) = x) &= \\ [\text{Prob}(N=0)]^{-1} \sum_{n=1}^{\infty} [\text{Prob}(L_\gamma(n, S) = x)] \text{Prob}(N=n) \\ &= 0, \text{ since } \text{Prob}(N=0) > 0.\end{aligned}$$

Proposition 4.

Type II = Prob ($TR \leq L_\gamma(N, S)$) $\leq 1 - \gamma$ for each $\alpha < 1$ and $T > 0$ where $M(T) = TR$.

Proof

Let $\alpha < 1$ and $T > 0$ with $M(T) = TR$.

Then

$$\begin{aligned} \text{Prob } (TR \leq L_\gamma(N, S)) &= \\ \text{Prob } (L_\gamma(N, S) = TR) + \text{Prob } (TR < L_\gamma(N, S)) &= \\ = \text{Prob } (TR < L_\gamma(N, S)), \text{ by Proposition 3,} & \\ = 1 - \text{Prob } (L_\gamma(N, S) \leq TR) \leq 1 - \gamma, \text{ by Proposition 2.} & \end{aligned}$$

Proposition 5.

For a growth curve with parameters $(\alpha, T, M(T))$, the expected number of failures ($E(N)$) can be determined by

$$E(N) = \frac{T}{(1-\alpha) M(T)}$$

Proof

The observed number of failures by test duration t , denoted by $N(t)$, is a non-homogeneous Poisson process with $N(T) = N$ and intensity function

$$\rho(t) = \frac{1}{M(t)} = \lambda \beta t^{\beta-1}$$

This implies that N is Poisson distributed with expected value

$$E(N) = \int_0^T \rho(t) dt = \lambda T^\beta$$

By Equation (18) in Section 3,

$$E(N) = \frac{T^\beta}{(M(T))^\beta T^{\beta-1}}$$

This yields

$$E(N) = \frac{T}{\beta M(T)} = \frac{T}{(1-\alpha) M(T)}.$$

Proposition 6.

For a growth curve with parameters $(\alpha, T, M(T))$,

$$\text{Prob}(A; \alpha, T, M(T)) =$$

$$(1-e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[\text{Prob} \left(\frac{\chi_{2n}^2}{z_\gamma^2(n)} \geq \frac{1}{2\mu d} \right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right)$$

where $\mu \triangleq E(N)$ and $d \triangleq M(T)/TR$.

Proof

From (23) in Section 3 and (33),

$$\begin{aligned} \text{Prob}(A; \alpha, T, M(T)) &= \text{Prob}(L_\gamma(N, S) \geq TR) \\ &= [1 - \text{Prob}(N=0)]^{-1} \sum_{n=1}^{\infty} [\text{Prob}(L_\gamma(n, S) \geq TR)] \text{Prob}(N=n) \\ &= [1 - \text{Prob}(N=0)]^{-1} \sum_{n=1}^{\infty} \left[\text{Prob} \left(\left(\frac{2T}{\beta} \right) \left(\frac{\chi_{2n}^2}{z_\gamma^2(n)} \right) \geq TR \right) \right] \text{Prob}(N=n) \\ &= [1 - \text{Prob}(N=0)]^{-1} \sum_{n=1}^{\infty} \left[\text{Prob} \left(\frac{\chi_{2n}^2}{z_\gamma^2(n)} \geq \frac{\beta(TR)}{2T} \right) \right] \text{Prob}(N=n) \end{aligned}$$

Letting $\mu \triangleq E(N)$ and $d \triangleq M(T)/TR$,

$$\begin{aligned} Prob(A; \alpha, T, M(T)) &= \\ (1-e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[Prob\left(\frac{\chi_{2n}^2}{z_\gamma^2(n)} \geq (1/2) \left(\frac{\beta M(T)}{T} \right) \left(\frac{TR}{M(T)} \right) \right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right) \\ &= (1-e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[Prob\left(\frac{\chi_{2n}^2}{z_\gamma^2(n)} \geq \frac{1}{2\mu d}\right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right). \end{aligned}$$

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STUDY GIST

SUBJECT: Technical Report No. XXX, Operating Characteristic Analysis for Reliability Growth Programs.

PRINCIPAL FINDINGS: The consumer and producer statistical risks associated with the MIL-HDBK-189 idealized growth curve can be determined from the expected number of failures and the ratio of the MTBF achieved at the end of test to the technical requirement to be demonstrated with confidence at the end of test.

MAIN ASSUMPTIONS: The system true reliability grows in accordance with the MIL-HDBK-189 idealized growth curve and the cumulative failure times are observed.

PRINCIPAL LIMITATIONS: The above assumptions are satisfied and the growth test is time-truncated.

SCOPE OF THE EFFORT: Methodology was developed to determine the a priori consumer and producer statistical risks of a reliability growth program.

OBJECTIVE: Extent concepts of an operating characteristic (OC) analysis for a reliability demonstration test to a reliability growth setting.

BASIC APPROACH: Obtain the probability distribution of the lower confidence bound for the system true reliability based on growth test data.

REASON FOR PERFORMING THE STUDY OR ANALYSIS: Develop methodology for assessing Government (consumer) and contractor (producer) statistical risks associated with a reliability growth program.

IMPACT OF THE STUDY: Allows analysts to perform trade-offs between test duration, consumer and producer statistical risks, and reliability growth parameters (i.e., growth rate and initial MTBF over initial test time period).

SPONSOR: AMSAA, RAM Division

PRINCIPAL INVESTIGATOR: Paul Ellner, AMSAA

NAME/ADDRESS/PHONE NUMBER WHERE COMMENTS & QUESTIONS CAN BE SENT:

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OTHER THAN SPONSOR, WHO COULD BENEFIT FROM THIS STUDY/INFORMATION?

DOD contractors and Government RAM analysts.